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Cryogenics 40 (2000) 469–474

Cryogenics

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Performance evaluation of counter flow heat exchangers considering the effect of heat in leak and longitudinal conduction for low-temperature applications

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Received 4 September 2000; accepted 13 November 2000

Abstract

Counter flow heat exchangers are commonly used in cryogenic systems because of their high effectiveness. In addition to operating and design parameters, the thermal performance of these heat exchangers is strongly governed by various losses such as longitudinal conduction through wall, heat in leak from surrounding, flow maldistribution, etc. Design based on conventional procedure, which normally does not take these losses into consideration, could be misleading and actual performance could be quite different than the predictions. In this paper, the numerical model developed earlier is extended to take into consideration the effect of heat in leak and the predictions are compared with the experimental results. The study is further extended to understand quantitative effect of heat in leak and axial conduction parameters on degradation of heat exchanger performance for 300–80 K and 80–20 K temperature range.

Keywords: Heat exchangers; Counter flow; Effectiveness

1. Introduction

Heat exchangers are one of the most critical components in any liquefaction/refrigeration system. Its effectiveness governs the efficiency of the whole system. The major requirement of these heat exchangers, working in the cryogenic temperature range, is to have high effectiveness [1]. In the recent past, Atrey [2] has shown in his analysis that decrease in heat exchanger effectiveness from 97% to 95% reduces the liquefaction by 12%. The design of heat exchangers, therefore, is very important from the system performance point of view. The design should take various losses, occurring during the exchange of heat, into consideration. The performance of the heat exchanger is governed by various parameters like mass flow rates, pressures and temperatures of working fluids, etc. Heat exchanger effectiveness takes into consideration the limitations of heat transfer between two heat exchanging streams due to these parameters. However, the performance is further deteriorated by various losses like longitudinal heat conduction across the wall material, heat in leak from the surrounding, flow maldistribution, etc., which are not normally taken into account while computing the heat exchanger effectiveness. Longitudinal heat conduction degrades the performance of the heat exchangers significantly for short flow length exchangers designed for the conditions of high effectiveness [3]. The performance may get further deteriorated due to heat in leak from surrounding. Researchers have developed closed-form solution for studying the longitudinal heat conduction through wall and heat leak from surrounding independently [4–7]. Recently, Gupta et al. [8] have published the results for coiled tube in tube heat exchangers obtained numerically and also experimentally for 300–80 K temperature range considering the longitudinal heat conduction through wall and gas property variations with temperature. The longitudinal heat conduction parameter was, however, not significant owing to small wall thickness of the tube. In the present paper, the numerical treatment applied to coiled tube-in-tube heat exchangers is further extended to take into account the effect of heat in leak in addition to the longitudinal heat conduction. The predictions of the model are then compared with the actual experimental results. The study is then extended to understand the effect of longitudinal heat conduction and heat in leak

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0011-2275/01/$ - see front matter
PII: S0011-2275(00)00069-2

Nomenclature

A	surface area of heat transfer (m^2)
A_c	cross-section area (m^2)
C	heat capacity rate of fluids defined by the product of m and C_p (W/K)
h	heat transfer coefficient (W/m^2 K)
k	thermal conductivity of the wall material (W/m K)
L	length of heat exchanger (m)
m	mass flow rate (kg/s)
n	number of transfer units for individual fluid stream
ntu	overall number of transfer units
Q_a	heat transfer from surrounding $= U_o A_o (T_a - T_c)$
R	dimensionless parameter defined as in Eq. (4)
U	overall heat transfer coefficient (W/m^2 K)
x	axial co-ordinate (m)
X	dimensionless axial co-ordinate in heat exchanger defined by x/L

Greeks

α	heat in leak parameter defined in Eq. (5)
θ	dimensionless temperature, defined as in Eq. (4)
ε	effectiveness of heat exchanger
λ	dimensionless longitudinal conduction parameter, defined as in Eq. (5)
μ	heat capacity rate ratio (C_h/C_c)
v	heat capacity rate ratio (C_{min}/C_h)
τ	degradation factor as defined in Eq. (11)

Subscripts

a	ambient
c	cold fluid
h	hot fluid
i	inner
o	outer
out	outlet
w	wall
in	inlet

parameters on the heat exchanger performance together or independently for gas-to-gas counter flow heat exchangers. Energy equations are formulated for hot and cold working fluids and wall. The equations are solved by applying finite difference techniques. Results are compared with ideal heat exchanger performance by defining degradation parameters. The variations in these parameters are studied for different ntu and heat capacity ratios.

1.1. Model formulation

Gupta et al. [8] have presented a numerical model consisting of longitudinal conduction and property variations of gas at low temperatures. The predictions of the model were compared with the experimental data obtained for different mass flow rates. It was observed that the predicted temperature profile towards the cold end of the heat exchanger was on a higher side as compared to the experimental results. One of the parameters responsible for this is the heat in leak from atmosphere which was not considered in the model formulation. Recently, Narayanan et al. [9] presented a mathematical model considering the heat loss through the wall at the cold end. This is mainly incorporated by taking a suitable boundary condition at the cold end of the wall. Gupta et al. [8] have taken an adiabatic boundary condition at the end for the tube-in-tube heat exchangers as the heat in leak due to conduction at the cold end of the wall is negligible due to small heat transfer area of the wall. Also, this could be isolated from the system by putting heat isolators in between making it a valid assumption. However, the heat in leak from the atmosphere to the cold fluid, flowing in the annular region of the tube-in-tube heat exchanger, is significant. It governs the temperature profile of the working fluids during the heat exchange. Considering these aspects, the model developed earlier is extended to account for this heat in leak.

The governing equations for the hot fluid, the cold fluid and the wall are as follows:

Hot fluid:

$$\frac{d\theta_h}{dX} + n_h(\theta_h - \theta_w) = 0. \tag{1}$$

Wall:

$$\lambda\gamma \frac{d^2\theta_w}{dX^2} + n_h(\theta_h - \theta_w) - \frac{n_c}{\mu}(\theta_w - \theta_c) = 0. \tag{2}$$

Cold fluid:

$$\frac{d\theta_c}{dX} + n_c(\theta_w - \theta_c) - \alpha\mu v(\text{ntu})\theta_c = -\alpha\mu v(\text{ntu})(R+1). \tag{3}$$

In the above expressions, θ_h, θ_w and θ_c represent the dimensionless temperature of hot fluid, wall and cold fluid, respectively, as given below. The following dimensionless parameters have been used for analysis.

$$\theta = \frac{t - t_{c,in}}{t_{h,in} - t_{c,in}}, \quad R = \frac{t_a - t_{h,in}}{t_{h,in} - t_{c,in}}, \quad X = \frac{x}{L}, \tag{4}$$

$$v = \frac{C_{min}}{C_h}, \quad \mu = \frac{C_h}{C_c}, \quad \alpha = \frac{U_o A_o}{U_i A_i}, \quad \lambda = \frac{kA_c}{C_{min} L}, \tag{5}$$

$$n_h = \left(\frac{hA}{C}\right)_h, \quad n_c = \left(\frac{hA}{C}\right)_c, \quad \text{ntu} = \frac{U_i A_i}{C_{min}}, \tag{6}$$

where n_h and n_c are the local heat transfer units of hot fluid and cold fluid streams, respectively; ntu is the total heat transfer unit. The heat in leak parameter (α) is defined as the ratio of external conductance (U_oA_o) to the internal conductance (U_iA_i) [4,5]. This parameter determines the amount of heat in leak from outside to the system.

The above equations are converted into linear algebraic equations by applying the finite difference method and are solved by Gauss–Joardon method. Assuming the adiabatic conditions for the wall at the two ends, the following boundary conditions are applied:

$X = 0$:

$$\theta_h = 1, \quad \frac{\partial \theta_w}{\partial X} = 0. \tag{7}$$

$X = 1$:

$$\theta_c = 0, \quad \frac{\partial \theta_w}{\partial X} = 0. \tag{8}$$

It is assumed that the local ntu of each fluid stream is the same; $n_h = n_c$ [7,9].

1.2. Effectiveness of the heat exchanger

The effectiveness of any heat exchanger is defined as the ratio of actual heat transfer to the maximum possible heat transfer. Because of the irreversibility in heat exchanger, the cold fluid does not take up total heat transferred by the hot fluid stream. As a result of this, the effectiveness of heat exchanger based on hot fluid and cold fluid are not equal. In the present paper, effectiveness is defined based on the hot fluid stream. It can be expressed mathematically as follows:

$$\varepsilon_h = \frac{q_{hot}}{q_{max}} = \frac{C_h(t_{h,out} - t_{h,in})}{C_{min}(t_{h,in} - t_{c,in})}. \tag{9}$$

The above expressions can be expressed in terms of dimensionless parameters as follows:

$$\varepsilon = (1 - \theta_{h,out})/v. \tag{10}$$

2. Degradation factor

Extending the definition of conduction effect factor given by Chiou [10] and Ranganayakulu [11], the degradation factor, τ, is defined to consider the deterioration in the performance of the heat exchanger due to heat in leak from surrounding and longitudinal conduction through wall. It is expressed mathematically in terms of effectiveness as follows:

$$\tau = \frac{\Delta\varepsilon}{\varepsilon} = \frac{\varepsilon_{NC,NHL} - \varepsilon_{WC,WHL}}{\varepsilon_{NC,NHLl}}, \tag{11}$$

where NC = no conduction, WC = with conduction, NHL = no heat in leak and WHL = with heat in leak.

It could be seen from (11) that τ is 0.0 if no losses are considered in the calculations of ε. τ increases with the consideration of losses in the calculation.

3. Results and discussion

3.1. Comparison of experimental results with the present model

Gupta et al. [8] presented a comparison of experimental results with the model predictions. This model did not consider the heat in leak in the system for the coiled tube in tube heat exchangers. The predictions of the model, based on the present analysis considering the heat in leak in the system from the atmosphere for the same heat exchangers, are compared with the actual results. Fig. 1 gives this comparison for the mass flow rate of 1.8 g/s in terms of temperature profile of warm and cold fluids across the length of the heat exchanger. It could be seen from the figure that the match between the actual results and the predictions based on the present model is better than those with the earlier model. Table 1 gives the details of the parameters used for present analysis. It is observed from the experimental results that the cold end of the heat exchanger gets more affected by the heat in leak from the surroundings. It is seen from the figure that the hot stream outlet temperature shows a better match with the actual temperature as compared with the earlier model. It is observed that the match for the warm end of the heat exchanger at a length of 2 m is not as good as the one predicted earlier. As clear from the formulation, the parameter α determines the amount of heat in leak from the atmosphere and is an empirically obtained parameter which is assumed to be constant across the length of the heat ex-

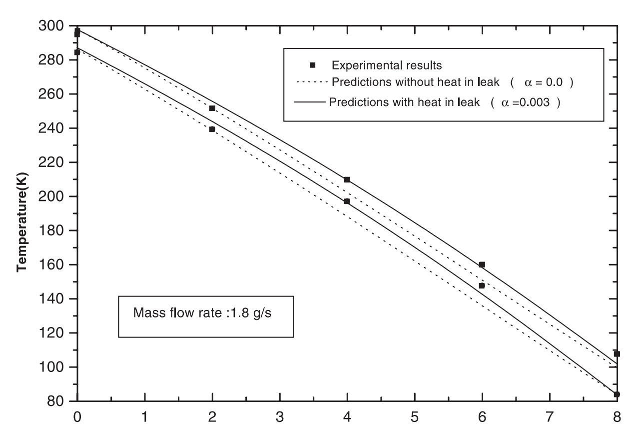

Fig. 1. Temperature profile of hot and cold fluid streams along the length of heat exchanger.

Table 1
Details of experimental parameters

Working fluid	Helium
Temperature range	300–80 K
Mass flow rate (g/s)	(1) $m_h = m_c = 1.8$ g/s (2) $m_h = m_c = 0.9$ g/s
Heat transfer surface area	0.160 m^2
Overall heat transfer coefficient	(1) 930 W/m^2 K. (2) 530 W/m^2 K
Longitudinal heat conduction parameter, λ	(1) 4.52×10^{-5}. (2) 9.01×10^{-5}
Number of transfer units, ntu	(1) 15.0; (2) 18.0
Ambient temperature	300 K
Heat in leak parameter, α	0.003

changer. However, it is possible that it may vary across the length also. The value of α has been taken equal to 0.003 to show the comparison. It should, however, be deduced accurately by conducting repeated experimental trials under various operating conditions.

To highlight the arguments quantitatively, Table 2 gives the temperature values across the length of the heat exchanger for (i) no heat in leak condition ($\alpha = 0$) (ii) heat in leak condition ($\alpha = 0.003$) and (iii) experimental results for helium mass flow rate of 0.9 g/s and ntu equal to 18. One can see the closeness of the match on the cold end of the temperature for $\alpha = 0.003$ with the actual results.

Once the significance of the analysis is established, the work is further extended to understand the effect of various operating parameters like capacity ratios, temperature conditions, etc., on the degradation of the heat exchangers for various ntu conditions and for various heat in leak and longitudinal heat conduction parameters.

3.2. Effect of heat capacity ratio (C_c/C_h) on degradation, τ

The performance of the heat exchanger is affected significantly by the heat capacity ratio. In the present analysis, its effect is studied for two cases. In the first case, hot stream enters at room temperature 300 K and cold stream enters at 80 K ($R = 0.0$) and in the second case, hot stream enters at 80 K and cold stream enters at 20 K ($R = 3.67$).

Figs. 2 and 3 show the effect of C_c/C_h on τ for $R = 0$ and for $R = 3.67$, respectively. The figures show the contribution towards degradation, τ, due to λ and α individually and also in a combined manner. It could be seen from these figures that the degradation curve due to λ alone goes through a maximum at $C_c/C_h = 1$. This is due to the fact that the temperature gradient across the length of the wall is maximum for this value. Similar trends have been reported in the literature [7,10,11]. It could be noted from the figures that the values of τ due to conduction alone does not change with the value of R.

The second curve in the same figures shows the degradation due to heat in leak, α, alone. For $R = 0.0$ and $\alpha = 0.001$, the degradation curve shows a maximum at $C_c/C_h = 1.0$. This is due to the fact that the heat in leak

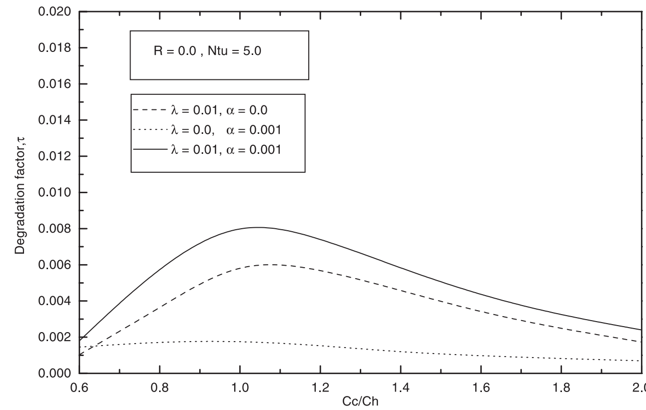

Fig. 2. Effect of C_c/C_h on τ for $R = 0.0$.

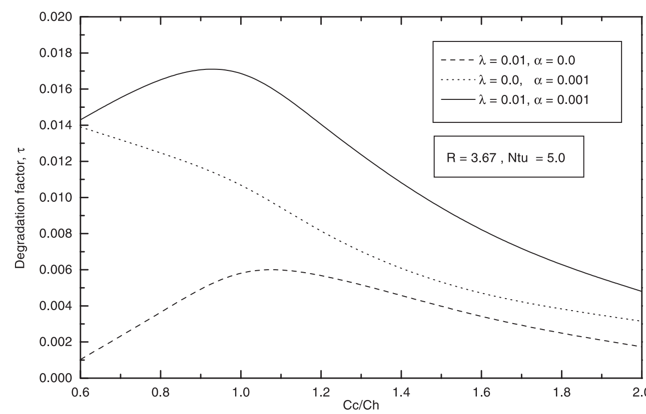

Fig. 3. Effect of C_c/C_h on τ for $R = 3.67$.

Table 2
Temperature distribution in coiled tube in tube heat exchanger, ntu = 18, $\lambda = 9.01 \times 10^{-5}$ and $R = 0.0$

x (m)	$\alpha = 0.0$		$\alpha = 0.003$		Experimental result	
	T_h (K)	T_c (K)	T_h (K)	T_c (K)	T_h (K)	T_c (K)
0.0	297.0	288.94	297.0	289.89	297.0	283.4
2.0	257.0	248.5	261.5	252.9	252.3	244.9
4.0	200.1	191.8	218.2	207.1	211.4	204.3
6.0	156.0	146.5	165.8	151.6	162.2	153.3
8.0	99.7	84.9	102.6	84.9	104.5	84.9

at this temperature level (300–80 K) is not sufficient enough to dominate the effect of imbalance in heat exchange between the hot and the cold streams. On the other hand, for $R = 3.67$ (80–20 K), the degradation curve shows a downward trend with the increase in C_c/C_h values. This is due to the fact that at lower temperature, heat exchanger experiences more heat flow from the surrounding. This heat inleak overrides the effect caused due to imbalanced flow in the heat exchanger. The degradation, however, is less for higher C_c/C_h ratios as higher C_c implies less temperature change for the cold fluid. It could be seen from Fig. 3 that for $R = 3.67$, the degradation due to heat in leak is significantly more than that due to longitudinal conduction. Such a situation can occur in tube-in-tube heat exchanger where longitudinal conduction loss is negligible. While, for $R = 0.0$, the loss due to conduction is comparable with that due to heat in leak.

The third curve in these figures shows an overall effect of both the losses together on the degradation factor. The nature of the curve depends on the individual degradation curves obtained for the given values of λ and α. The degradation is maximum for $C_c/C_h = 1$ and above and below this value it shows a decrease.

3.3. Effect of heat in leak, α, on degradation, τ

Figs. 4 and 5 show variation of τ with ntu for different heat in leak conditions, i.e. different α values, for $R = 0$ and 3.67, respectively. The effect is shown for different C_c/C_h values and for $\lambda = 0.05$. It could be observed from Fig. 4 that for $\alpha = 0.0$ (i.e. no heat leak from surrounding), degradation curve reaches a maximum for ntu = 6 and after that it decreases. Also, the degradation is maximum for $C_c/C_h = 1.0$. These findings are in accordance with the results reported in the literature [7,10,11]. It has been seen that the quantitative values of the degradation obtained are close to those reported by Kroger [7].

As the value of α increases, the maximum obtained in earlier curve disappears and the degradation increases with ntu. If α is on a higher side, the slope of the degradation curve increases. It could be observed that for higher ntu heat exchangers, the advantage of higher ntu could be lost due to more degradation and this fact has to be realized at the design stage only.

Fig. 5 shows that the nature of the curves remains the same at lower temperature, i.e. $R = 3.67$. The degradation curve for $\alpha = 0$ remains the same as it is for $R = 0$. However, for higher α, the degradation is quite severe as compared to those reported for $R = 0.0$. For example, for $R = 3.67$, and $\alpha = 0.0005$ the degradation is 5%, for ntu = 20 and $C_c/C_h = 1$. On the other hand, for $R = 0.0$, the degradation is only 2% for the same case.

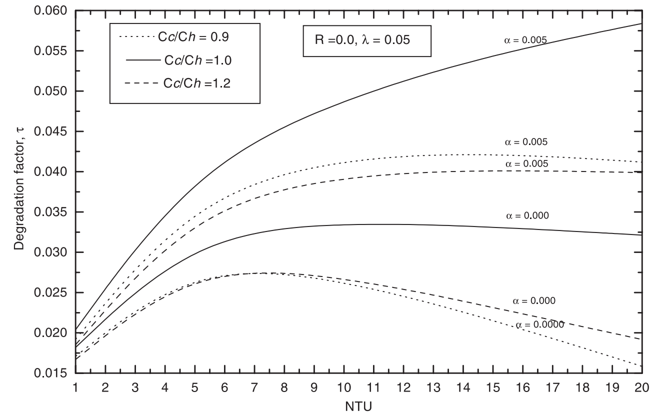

Fig. 4. Effect of ntu on τ for different C_c/C_h and $R = 0.0$.

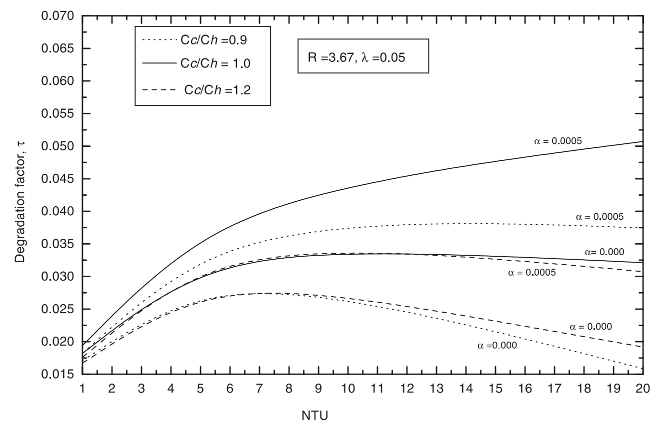

Fig. 5. Effect of ntu on τ for different C_c/C_h and $R = 3.67$.

3.4. Effect of longitudinal heat conduction, λ, and heat in leak, α, on τ

Fig. 6 shows the effect of λ and α on τ for $C_c/C_h = 1$, and for $R = 0$. The figure shows the increase in degradation when λ and α are increased from 0.05 to 0.1 and from 0 to 0.005, respectively. Figs. 4 and 5 show that if

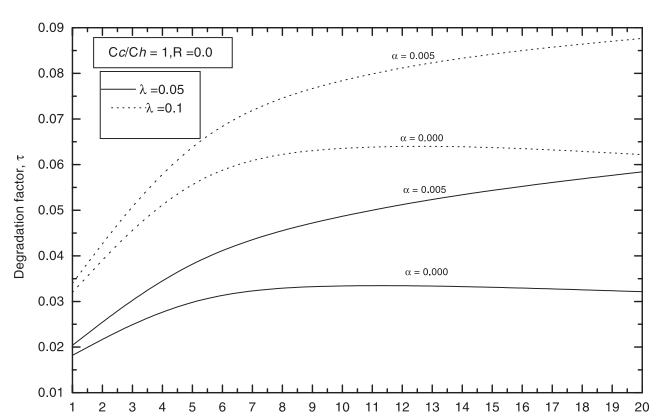

Fig. 6. Effect of ntu on τ for different λ and α parameters.

C_c/C_h is more than or less than unity, a distinct maximum deterioration could be observed at a particular value of ntu. However, for very high values of α, this may not be so and the degradation may continue to increase till a higher value of ntu. Similar trends are expected for low temperature, i.e. $R = 3.67$, however, with increased magnitude of degradation in comparison with those at $R = 0$.

4. Conclusions

The following conclusions could be drawn based on the results presented above.

The performance of a heat exchanger, apart from operating parameters, depends on various factors such as heat in leak, longitudinal conduction, operating temperature level, etc. The comparison of the predictions with the experimental results confirm the importance of heat in leak parameter and longitudinal heat conduction. In the present case of coiled tube in tube heat exchangers, heat in leak parameter, α, was found to be around 0.003 for the temperature range of 300–80 K. It is observed that the degradation in performance is severe for low-temperature application especially when α is significant. The degradation is maximum for the balanced flow condition which can therefore be a conservative design criterion for any heat exchanger. The study also makes it clear that increasing ntu can cause more degradation due to heat in leak and this has to be weighed correctly at the design stage only. The graphical representation of the results, presented above, can serve as useful guidelines for designing a heat exchanger for cryogenic applications.

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